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Absolutely (p, q)-summing multiplier sequences and functions with applications to tensor products

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Arregui and Blasco (J. Math. Anal. Appl. 274 (2002)) introduced the concept of (p, q)-summing multiplier and (p, q)-summing sequence and discussed several interesting applications in the theory of Banach spaces. Let X and Y be Banach spaces and let $1 \leq p, q \leq \infty$. A sequence $(u_j)_{j\in\mathbf{N}}$ in $\mathcal{L}(X, Y)$ (bounded linear operators) is a (p, q)-summing multiplier for the pair X, Y – indicated by $(u_j) \in (\ell_q^{weak}(X), \ell_p(Y))$ or $(u_j) \in \ell_{\pi_{p,q}}(X, Y)$ – if there exists a constant $C > 0$ such that, for any finite collection of vectors $x_1, x_2, \ldots, x_n$ in X, it holds that

$$(\sum_{j=1}^{n} \|u_j x_j\|^p)^{1/p} \leq C \sup \left\{ (\sum_{j=1}^{n} |x^* x_j|^q)^{1/q} : \|x^*\| \leq 1 \right\}.$$

A bounded sequence $(x_n) \subset X$ such that $(x_n) \in \ell_{\pi_{p,q}}(X^*, \mathbf{K})$ is called a (p, q)-summing sequence in X; the space of all these sequences is denoted by $\ell_{\pi_{p,q}}(X)$. A similar definition holds for the space $(\ell_q(X), \ell_{\pi_{1,p'}}(X))$ (where $\frac{1}{p} + \frac{1}{p'} = 1$) of strongly (p, q)-summing multipliers. We discuss several special examples and properties of (p, q)-summing multipliers and strongly (p, q)-summing multipliers; we also consider some duality results and applications of (p, q)-summing sequences to characterizations of projective tensor products of some Banach spaces. A short discussion of a possible similar theory in the context of function spaces will conclude our discussion.