

Numerical construction of generalized Hermite polynomials

Aleksandar S. Cvetković, Niš, Serbia

In this lecture we presented results, from [3], obtained jointly with G.V. Milovanović.

Inspired by Ball's paper [1], we consider numerical construction of the three term recurrence coefficients for the generalized Hermite polynomials. By the generalized Hermite polynomials, we understand polynomials orthogonal with respect to the weight function

$$w(x; z) = |x - z|^\gamma e^{-x^2}, \quad z \in \mathbb{R}, \quad \gamma > -1,$$

supported on the real line.

With an extensive use of computer algebra, *Mathematica* package, we are solving for the series in n representing three term recurrence coefficients. Our starting point are the so called Laguerre-Freud nonlinear recurrence relations for the three term recurrence coefficients (see [2]). We are discussing numerical stability of Laguerre-Freud nonlinear recurrence relations as z changes.

Finally, we are presenting some numerical algorithms which can be applied successfully for the numerical construction of the three term recurrence coefficients.

Also, we are improving result presented in [1], adding few more terms in the approximation of the three term recurrence coefficients, which extremely reduces the complexity of the algorithm presented in [1], for the construction of the three term recurrence coefficients for the polynomials orthogonal with respect to the weight function

$$w(x) = x^\gamma e^{-x^2} \chi_{\mathbb{R}}, \quad \gamma > -1.$$

References

- [1] J. S. Ball, *Half-range generalization Hermite polynomials and the related Gaussian quadratures*, SIAM J. Numer. Anal. **40** (6) (2003), 2311–2317.

- [2] A. P. Magnus, *Painlevé-type of differential equations for the recurrence coefficients of semi-classical orthogonal polynomials*, J. Comput. Appl. Math. 57 (1995) 215–237.
- [3] G. V. Milovanović, A. S. Cvetković, *Numerical construction of generalized Hermite polynomials*, (submitted).