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Uniform integrability and embeddings

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A classical theorem of de Vallée Poussin characterizes the uniformly integrable subsets of $L_1(X, \mu)$, where (X, μ) is a probability space, as those sets that are bounded in some Orlicz space L_Φ, with $\lim_{t\to\infty} \frac{\Phi(t)}{t} = \infty$. In this paper we investigate the possibility of replacing the Orlicz space in this result by other types of Banach function spaces.

Call a family $\mathcal{E}$ of Banach spaces of integrable functions *adequate* if a subset K of $L_1(X, \mu)$ is uniformly integrable if and only if there is an $E \in \mathcal{E}$ such that K is a bounded subset of E. We show that adequate families of rearrangement invariant Banach function spaces can be characterized in terms embeddings, and in terms of the fundamental functions of these spaces. These general results make it possible to recover and to improve on de Vallée Poussin's theorem.

The fact that uniform integrability can be characterized in terms of rearrangements of functions make it possible to extend some of these results to the context of non-commutative function spaces. If $\mathcal{M}$ is a von Neumann algebra, we may regard it as a non-commutative L_∞ space, and its predual as a non-commutative L_1 space. If $\mathcal{M}$ is semi-finite, it admits a faithful normal semi-finite trace. It is possible to define the notions of a measurable operator and its generalized singular functional (the analogue of the rearrangement of a measurable function) relative to such a trace. This makes it possible to introduce non-commutative versions of classical reaarangement invariant function spaces. We explore characterizations of uniform integrability in this context.