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Some optimal quadrature rules of probabilistic type

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Recently, in [1], the authors obtained the following Ostrowski inequality for random variables, where, for $p \geq 1$, q stands for the conjugate of p, I is a real interval and $\mathcal{W}_p$ is the space of locally absolutely continuous functions $f : I \to R$ such that $f' \in L_p(I)$.

Theorem 1 *Let X and Y be two I-valued random variables having distribution functions F and G, respectively, and let $1 \leq p \leq \infty$. If $F - G \in L_q(I)$, then, for each $f \in \mathcal{W}_p$ such that $f(X)$ and $f(Y)$ are integrable, we have*

$$|Ef(Y) - Ef(X)| \leq \|F - G\|_q \|f'\|_p. \tag{1}$$

In this work, we keep fixed the random variable Y and using inequality (1), for $f \in \mathcal{W}_p$ such that $f(Y)$ is integrable, we obtain quadrature rules to approximate the integral

$$Ef(Y) = \int_I f\, dG.$$

We show these rules are optimal in certain sense and give some applications of our results which include the uniform and the exponential distribution. The Laplace transform of a function is also studied.

References

[1] J. de la Cal, J. Cárcamo, *A Ostrowski-type inequality for random variables with applications,* submitted for publication, 2004.