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Vector lattices of disjointness preserving operators

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Let $\mathcal{L}_b(L,M)$ be the ordered vector space of all order bounded (linear) operators between two Archimedean vector lattices L and M. A subset $\mathcal{D}$ of $\mathcal{L}_b(L,M)$ is said to be a δ-*subset* if $x \perp y$ in L implies $Sx \perp Ty$ in M for all $S,T \in \mathcal{D}$. It turns out that, for any δ-subset $\mathcal{D}$ of $\mathcal{L}_b(L,M)$, the generalized vector sublattice $\mathcal{L}(\mathcal{D})$ of $\mathcal{L}_b(L,M)$ generated by $\mathcal{D}$ can be defined. This seems to be rather surprising since $\mathcal{L}(\mathcal{D})$ need not exist for an arbitrary subset $\mathcal{D}$ of $\mathcal{L}_b(L,M)$, obviously. Moreover, if $\mathcal{D}$ is a δ-subset of $\mathcal{L}_b(L,M)$ then $\mathcal{L}(\mathcal{D})$ is a δ-subset of $\mathcal{L}_b(L,M)$ on its own. In particular, all elements in $\mathcal{L}(\mathcal{D})$ are disjointness preserving. As an application, we constructively and intrinsically prove that the modulus of an order bounded disjointness preserving operator on Archimedean complex vector lattices exists. We then introduce the notion of maximal (with respect to the inclusion) δ-subsets of $\mathcal{L}_b(L,M)$ as a generalization of the collection $\mathrm{Orth}(L)$ of all orthomorphisms on L. In this regard, we prove that any maximal δ-subset of $\mathcal{L}_b(L,M)$ is a generalized vector sublattice of $\mathcal{L}_b(L,M)$. Several other well-know properties of $\mathrm{Orth}(L)$ are extended to maximal δ-subsets of $\mathcal{L}_b(L,M)$, most of them are illustrated by a complete description of maximal δ-subsets of order bounded operators on Banach lattices of continuous functions.