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Bernstein-Durrmeyer type quasi-interpolants on intervals

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Basic Bernstein polynomials of degree $n \in \mathbf{N}$ on $I = [0,1]$ are given by

$$B_\alpha(x) = \binom{n}{\alpha_1} x^{\alpha_1}(1-x)^{\alpha_0}, \quad \alpha_1, \alpha_0 \in \mathbf{N}_0, \quad |\alpha| := \alpha_1 + \alpha_0 = n.$$

For $\mu_1, \mu_0 > -1$ we define the inner product $< f, g >_\mu := \int_0^1 x^{\mu_1}(1-x)^{\mu_0} f(x)g(x)dx$. The classical Bernstein-Durrmeyer operator is defined as

$$M_{n,\mu}(f) := \sum_{|\alpha|=n} \frac{< f, B_\alpha >_\mu}{< 1, B_\alpha >_\mu} B_\alpha, \quad f \in L[0,1].$$

This operator is very well studied by Derriennic, Berens, Xu, Ditzian, Chen, Ivanov, among others.

Recently, Jetter and Stöckler introduced the differential operators

$$\mathcal{U}_{\ell,\mu} := \frac{(-1)^\ell}{(\ell!)^2} \frac{1}{x^{\mu_1}(1-x)^{\mu_0}} \frac{d^\ell}{dx^\ell} \left(x^{\mu_1+\ell}(1-x)^{\mu_0+\ell} \frac{d^\ell}{dx^\ell} \right), \quad \ell = 0, 1, ...,$$

and the quasi-interpolant operators of type (r, n), $0 \le r \le n$,

$$M_{n,\mu}^{(r)}(f) := \sum_{\ell=0}^{r} \frac{1}{\binom{n}{\ell}} \mathcal{U}_{\ell,\mu}(M_{n,\mu}(f)).$$

In particular, $M_{n,\mu}^{(0)}$ coincides with the classical Bernstein-Durrmeyer operator $M_{n,\mu}$.

We study spectral properties of the operators $\mathcal{U}_{\ell,\mu}$ and $M_{n,\mu}^{(r)}$. As a consequence, we obtain a Voronoskaja type result for the operators $M_{n,\mu}^{(r)}$ and a Jackson-Favard type error estimate. Finally, we discuss the problem of estimating the approximation order in terms of the adequate K-functional.

Joint work with K. Jetter and J. Stöckler.