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The small ball property in Banach spaces (quantitative results)

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A metric space (M, d) is said to have *the small ball property (sbp)* if for every $\varepsilon_0 > 0$ it is possible to write M as a union of a sequence $(B(x_n, r_n))$ of closed balls such that the r_n are smaller than ε_0 and $\lim r_n = 0$.

In joint work with V. Kadec from Charkow this property has been investigated systematically, the main results will be reviewed at the beginning of the talk:

1. Bounded convex closed sets in Banach spaces have the *sbp* only if they are compact.

2. Precisely the finite dimensional Banach spaces have the *sbp*.

3. Let B be a boundary in the bidual of an infinite-dimensional Banach space. Then B does not have the *sbp*. In particular the set of extreme points in the unit ball of infinite dimensional reflexive Banach spaces fails to have the *sbp*.

Then we will turn to *quantitative results*. We will investigate the collection of sequences (r_n) for which sequences (x_n) exist such that the union of the $(B(x_n, r_n))$ covers M. Also we assign to M the infimum of the ε_0 with the above *sbp*-property. Surprisingly, for many natural subsets of Banach spaces this infimum is either zero (which corresponds to the *sbp*-case) or one.