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On existence of functions with prescribed norms of its derivatives

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In this talk we shall discuss the following problem which was posed by Kolmogorov:

For given integer d, given numbers M_{ν_i}, $1 \le \nu_i \le r$, $1 \le i \le d$ and function space X find necessary and sufficient conditions for existence $x \in X$ such that

$$\left\| x^{(\nu_i)} \right\|_\infty = M_{\nu_i}.$$

We shall give a short review of known results and present new ones. In particular, we will give a complete characterization of sets of four and five numbers such that there exists l-monotone function with prescribed smoothness that has these numbers as values of sup-norms of its corresponding derivatives. To be more precise we need the following notation.

We shall denote by $L_p^r(\mathbb{R}_-)$, $\mathbb{R}_-$ is negative half-line, $r \in \mathbb{N}$, the space of functions $x : \mathbb{R}_- \to \mathbb{R}$, that have locally absolutely continuous derivative $x^{(r-1)}$ and satisfy $x^{(r)} \in L_p(\mathbb{R}_-)$. For $1 \le p, s \le \infty$ let $L_{p,s}^r(\mathbb{R}_-) = L_s^r(\mathbb{R}_-) \cap L_p(\mathbb{R}_-)$. As $L_{p,s}^{r,l}(\mathbb{R}_-)$ for $l < r, l \in \mathbb{N}$ denote the class of functions $x \in L_{p,s}^r(\mathbb{R}_-)$ that are non-negative with all its derivatives up to and including order l.

New results solve Kolmogorov problem in the following cases:

1. $X = L_{\infty,\infty}^{r,r-2}(\mathbb{R}_-)$; $\quad d = 4$, $\nu_1 = 0, \nu_2 = k, \nu_3 = l(l > k), \nu_4 = r$.
2. $X = L_{\infty,\infty}^{r,r-1}(\mathbb{R}_-)$; $\quad d = 5$, $\nu_1 = 0, \nu_2 = k, \nu_3 = l(l > k), \nu_4 = r - 1, \nu_5 = r$.