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# Estimates of solutions for some classes of second order differential equations in Hilbert space

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In Hilbert space $H$ consider a second-order differential equation

$$u''(t) + (B + iD)u'(t) + (T + iS) = 0, \quad t \geq 0 \tag{1}$$

under the following conditions ($I$ is identity operator)

a) $T$ is linear selfadjoint positive defined operator

- $B^* = B \geqslant mI$ $(m > 0)$, $D$ is closed symmetric. $B$ and $D$ are $T^{\frac{1}{2}}$-bounded operators.
- $S$ is closed symmetric $T$-bounded operator, operator $(T + iS)$ is boundary invertible.

By $H_s$ denote a scale of Hilbert spaces generating by operator $T^{\frac{1}{2}}$ (i.e. $H_s$ is the domain of $T^{s/2}$ supplying the norm $\|x\|_s = \left\|T^{s/2}x\right\|$). For $a < m$ consider a quadric form

$$q_{a,\delta}(x) = 4\delta a\|x\|_1 - \big((B - aI)^{-1}(S - aD)x, (S - aD)x\big), \ x \in H_1$$

**Theorem 1** *Let $q_{a,\delta}(x) \geq -c\,\|x\|_1$ ($\forall x \in H_1$) for some $a \in (0, m)$, $\delta > 0$ and $c \in \mathbb{R}$. Then for all $u_0 \in H_2$, $u_1 \in H_1$ there exists a unique solution of the differential equation with initial data*

$$u(0) = u_0, \quad u'(0) = u_1$$

*and for some positive $M$ the inequality*

$$\|u(t)\|_1^2 + \|u'(t)\|^2 \le Me^{2\omega t}\left\{\|u_0\|_1^2 + \|u_1\|^2\right\}$$

*where*

$$\omega = \max\left\{\frac{c}{4\delta},\ (\delta - 1)m\right\}$$

In the talk there will be given some applications to the theory of partial differential equations.

## References

[1] Artamonov Nikita, *Estimates for solutions of some classes of second order differential equations in Hilbert space,* Math. Sbornik N193(8) (2003), 3–12.