FAAT $^2 0_4$

Intersections of Fréchet Spaces and (LB)–spaces

A. Albanese (speaker), *Lecce*, *Italy*

J. Bonet, *Valencia*, *Spain*

AMS Classification: 46A04

Keywords and phrases: Intersection spaces, Fréchet spaces, (LB)–spaces

We present results about the class of locally convex spaces which are defined as the intersection $E \cap F$ of a Fréchet space F and a countable inductive limit of Banach spaces E. They appear in several parts of analysis whenever the space F is determined by countably many necessary (e.g. differentiability of integrability) conditions and E is the dual of such a space, in particular E is defined by a countable sequence of bounded sets which may also be determined by concrete estimates. Two natural topologies can be defined on $E \cap F$: the intersection topology, which has seminorms easy to describe and which permits direct estimates, and a finer inductive limit topology which is defined in a natural way and which has good locally convex properties, e.g. $E \cap F$ with this topology is a barrelled space. It turns out that the locally convex properties of $E \cap F$ with the intersection topology are related to the completeness of the (LF)–space $E + F$.

This research continues the one in [2], where the case when F is a Fréchet Schwartz space (an (FS)–space) and E is the dual of an (FS)–space (an (DFS)–space) was analyzed. One the original motivations for the research in [2] came up in the investigations of Langenbruch [3,4,5] about the surjectivity of linear partial differential operators $P(D)$ with constant coefficients on spaces of ultradifferentiable functions and distributions. On the other hand, the investigations in [1] about the relation of the hypoellipticity of a linear partial differential operator with variable coefficients with the local solvability of its transpose in the setting of Gevrey classes required to consider when the intersection of a Banach space and a (DFS)–space is barrelled. This is one of the reasons why we study a setting which is more general

than the one of [2]. Moreover the intersections of function spaces appear also in other areas of functional analysis. It turned out that the locally convex structure of the intersection of two Fréchet spaces was rather intricate. Indeed, Taskinen [6] showed that the strong dual of the spaces $C(\mathbf{R}) \cap L^1(\mathbf{R})$ is not a (LB)–space.

References

[1] A.A. Albanese, A. Corli, L. Rodino, *Hypoellipticity and local solvability in Gevrey classes,,* Math. Nachr. 242 (2002), 5–16.

[2] J. Bonet, *Intersections of Fréchet Schwartz spaces and their duals,,* Arch. Math. 68 (1997), 320–325.

[3] M. Langenbruch, *Continuation of Gevrey regularity for solutions of partial differential operators, in:,* Functional Analysis, Walter de Gruyter, Berlin–New–York (1996), 249–280.

[4] M. Langenbruch, *Surjectivity of partial differential operators on Gevrey classes and extensions of regularity,* Math. Nachr. 196 (1998), 103–140.

[5] M. Langenbruch, *Surjective partial differential operators of ultradifferentiable functions of Roumieu type,* Resultate Math. 29 (1996), 254–275.

[6] J. Taskinen, *Examples of non–distinguished Fréchet spaces,* Ann. Acad. Sci. Fenn. Ser. A I Math. 14 (1989), 327–346.